

Chapter 9 — Ratios: Part-to-Part and Part-to-Whole — Teacher Guide

Chapter at a glance

Lesson duration	4 lessons, 45–60 minutes each (see pacing table below)
Materials	Red and blue counters, linking cubes, labeled quantity cards, tape-diagram strips, ratio tables, fraction cards, context cards, the three chapter diagrams
Launch question	“A group has 2 boys and 3 girls. Is boys:girls = 2:3 the same statement as boys:total = 2:5?”
Key representation	The labeled tape diagram — keep one visible throughout instruction
Anticipated responses	Correct: adds ratio parts to find the whole before writing a part-to-whole fraction. Partial: computes correctly but can’t explain why the parts must be added. Incorrect: treats a part-to-part ratio as if the second number were already the whole (for example, “boys are $\frac{2}{5}$ of the class”).
If the readiness check fails	Reteach fraction-as-relationship and equivalent fractions from Chapter 2 before starting the Launch; do not skip the labeled counters step
Talk prompt	“Are you comparing one part with another part, or a part with the whole?”
Exit ticket	5 items — part-to-whole fraction, ratio-with-total split, simplify, explain the part-to-part trap, state a check (see Section 14)
Reteach / extend	Reteach: ratios with small, familiar totals only, using physical counters. Extend: missing-total problems, multiple contexts for the same ratio.

Purpose and pacing

Chapter 9 begins the ratio and rate section. It establishes the language and structure needed for equivalent ratios, rates, and best-buy problems. The main teaching priority is not speed; it is preserving meaning and order.

Lesson	Focus	Suggested time
Lesson 1	Readiness, launch, part-to-part, and part-to-	45–60 minutes

Lesson	Focus	Suggested time
	whole	
Lesson 2	Tape diagrams, ratio with a total, and one ratio unit	45–60 minutes
Lesson 3	Simplifying ratios, order, misconceptions, and Worksheet A	45–55 minutes
Lesson 4	Mixed practice, Tripwire, Exit Ticket, and targeted reteaching	50–65 minutes

Materials

Prepare red and blue counters, linking cubes, labeled quantity cards, tape-diagram strips, ratio tables, fraction cards, and context cards. Keep color or symbol labels on all materials so students do not detach numbers from quantities.

Teacher launch guidance

Begin with the 2 boys and 3 girls context. Ask students to write as many true comparison statements as possible. Then classify them as part-to-part or part-to-whole. This makes the distinction meaningful before formal definitions are introduced.

When a student writes for the fraction of boys, ask the student to point to the three in the denominator. If the three means girls, it cannot represent the entire class. Return to the total of five parts.

Teacher prompts

Purpose	Prompt
Preserve order	“Which quantity is named first?”
Identify comparison type	“Are you comparing one part with another part, or a part with the whole?”
Find total parts	“What does the complete group contain?”
Find one unit	“How many equal ratio units fit in the total?”
Check a split	“Do the parts add back to the total?”
Simplify	“What common factor scales both quantities?”
Diagnose an error	“Which label or total did the student use incorrectly?”

Differentiation

Students needing concrete support

Use colored counters and physically build every ratio. Do not remove labels. Move from 2:3 and 1:4 to less familiar ratios only after students can state both the part-to-part and part-to-whole meanings.

Students needing language support

Use the frames:

- “The ratio compares _____ with _____.”
- “The first number represents _____.”
- “The total number of parts is _____.”
- “The fraction of the whole is _____ because _____.”
- “One ratio part is _____, so the total quantities are _____.”

Students ready to extend

Ask students to construct multiple contexts for the same ratio, compare ratios with the same part-to-whole fraction, solve missing-total problems, and explain why ratios can be equivalent even when their numbers differ.

Formative assessment

Record whether students preserve order, identify comparison type, add ratio parts, find one unit, calculate quantities, and check by recombining. A correct numerical split with reversed labels should be marked developing because later ratio and rate work depends on order.

Revision Support: Expected Ideas and Teacher Moves

Use this compact guide with the new Cumulative Connection and Strategy Studio tasks. Accept equivalent representations when the relationship, units, and reasoning are correct.

Cumulative Connection: Interpret 3:5 as a part-to-part ratio. Convert 3:5 into a part-to-whole fraction for the first part. Explain why 3:5 and 6:10 are equivalent. Write a ratio with total 24 and first part 9.

Strategy Studio guidance: Expected ideas: rice:vegetables = 3:5, rice:total = 3:8, vegetables:total = 5:8; 6:10 uses factor 2; 12:8, 8:12, and 12:20 are all valid with labels; equivalence requires a common multiplicative factor, not a common additive change.

Transfer Bridge Teacher Guidance

Use the four tasks to distinguish imitation from transfer. Ask students to name the relationship, keep units visible, compare representations, and explain or repair an error. Accept equivalent methods when the structure and interpretation are correct.

Expected guidance: Familiar: rice:vegetables = 3:5, rice:total = 3:8, vegetables:total = 5:8. Altered: 3:5 scaled by is 1.5:2.5. Non-routine: 12:8, 8:12, and 12:20 are valid when labels are stated. Repair: equivalence requires the same multiplicative factor on both parts; 3:5 becomes 6:10, not 6:8.

Record whether the difficulty is classification, representation, operation, computation, units, interpretation, or checking. Do not assign more routine practice until the source of the error is identified.

Discussion and evidence note

Ask students to name the relationship before calculating, compare at least two representations when appropriate, and revise an explanation after hearing another

strategy. Record whether the difficulty is classification, representation, operation, computation, units, or checking.

Answer Key

Ready-to-Learn Check

- Two out of five equal parts or a comparison of 2 with a whole of 5.
- 5.
- .
- 5 counters.
- They compare different first and second quantities.
- Flour.
- The complete amount made from all the ratio parts.

Worksheet A

- 5.** Likely wrong answer: 4 (uses only the second number, 4, as if it were the total). Misconception: treating the second ratio term as the whole instead of adding both parts. Next move: "how many total parts make up the whole group?"
- . Likely wrong answer: (uses the second ratio term as the denominator instead of the sum of both parts). Misconception: same as #1. Next move: "1 out of how many total parts?"
- 5.** Likely wrong answer: 3 (uses only the second number as the total). Misconception: same recurring error. Next move: "add both parts — what is the total?"
- . Likely wrong answer: (uses boys as the denominator instead of the total). Misconception: treating a part-to-part ratio as if one part were already the whole. Next move: "3 out of how many total parts?"

- Total parts 5; one part 5; boys 10.** Likely wrong answer: uses only 3 (the second ratio term) as the total, giving $25 \div 3$ and a messy, incorrect result. Misconception: not adding both ratio parts before dividing. Next move: "how many equal parts is the group of 25 split into?"
- Total parts 8; one part 5; dogs 25.** Likely wrong answer: uses only 5 (the second ratio term) as the total, giving $40 \div 5 \times 3 = 24$ instead of the correct 25. Misconception: same as #5. Next move: "add 3 and 5 first — how many total parts?"
- 2:3.** Likely wrong answer: 2:9 (divides only the first part by 3, leaving the second part unchanged). Misconception: scaling only one quantity instead of both. Next move: "what happened to the second number when you simplified?"
- Order identifies which quantity is first and which is second.** Common flaw: says "order matters" without explaining what specifically changes when it's reversed. Next move: ask for a concrete example where reversing the order changes the meaning.
- Two boxes labeled first quantity and three boxes labeled second quantity.** Common flaw: draws five boxes but doesn't label which color or group each box represents. Next move: require labels on every box, not just a count.
- A part-to-part ratio does not use the whole as its denominator; add the parts first.** Common flaw: states the rule without connecting it to why skipping this step gives a wrong fraction. Next move: ask what

denominator the student would get by skipping this step, and why it's wrong.

Worksheet B

1. **and** . Likely wrong answer: *and* (uses one ratio part as the denominator for the other, instead of the total). Misconception: treating a part-to-part ratio as if it were already two part-to-whole fractions. Next move: "3 out of how many total parts? 5 out of how many?"
2. **and** . Likely wrong answer: *and* (same reversed-denominator error as #1). Misconception: same as #1. Next move: same check.
3. **Total parts 12; one part 4; amounts 20 and 28.** Likely wrong answer: uses only one ratio number (5 or 7) as the total instead of adding them, giving a messy or incorrect split. Misconception: not adding both ratio parts before dividing. Next move: "how many total parts does 5:7 make?"
4. **Total parts 9; one part 7; amounts 28 and 35.** Likely wrong answer: same total-parts omission as #3. Misconception: same. Next move: same check.
5. **3:4.** Likely wrong answer: 9:12 (divides both parts by 2 instead of the full common factor 6, leaving the ratio only partially simplified). Misconception: stopping at a smaller common factor instead of the greatest one. Next move: "is there a larger number that still divides both 18 and 24 evenly?"
6. **5:8.** Likely wrong answer: applies a different factor to each part (for example, dividing 25 by 5 but 40 by 10), producing a distorted ratio. Misconception: not using one common factor for both parts. Next move: "does the same number divide both 25 and 40?"
7. **Total parts are 7, so boys are , not .** Common flaw: identifies that is wrong but doesn't explain why or supply the corrected fraction. Next move: require both the corrected fraction and the reasoning.
8. **Total parts 7; concentrate .** Likely wrong answer: (uses the water part as the denominator instead of the total). Misconception: same reversed-denominator error as #1. Next move: "3 out of how many total parts?"
9. **Total parts 7; one part 5; cats 25 and dogs 10.** Likely wrong answer: swaps the results, reporting 10 cats and 25 dogs. Misconception: losing track of which ratio number corresponds to which label. Next move: "does the first number in cats:dogs describe cats or dogs?"
10. **Answers vary but must contain equal-size boxes.** Common flaw: draws boxes of noticeably different sizes, which no longer represent equal ratio units. Next move: check that every box is the same size before checking the count.
11. **Yes, multiply 2:3 by 5.** Likely wrong answer: "No, because the numbers are different," without checking for a common scale factor. Misconception: assuming different-looking ratios can't be equivalent. Next move: "does one common factor turn 2:3 into 10:15?"
12. **The first and second quantities switch positions.** Common flaw: says "it changes" without stating specifically that the first and second quantities swap roles. Next move: ask the student to write both the original and reversed statement side by side.

Worksheet C

1. . Likely wrong answer: (uses the water part as the denominator instead of the total). Misconception: the recurring reversed-denominator error. Next move: "1 out of how many total parts?"
2. **Total parts 7; one part 8; Grade 7 = 24.** Likely wrong answer: uses only 4 (the second ratio number) as the total, giving a messy or incorrect split. Misconception: not adding both ratio parts before dividing. Next move: "how many total parts does 3:4 make?"
3. **Total parts 7; one part 4 L; white = 20 L.** Likely wrong answer: uses only 5 (the second ratio number) as the total. Misconception: same as #2. Next move: same check.
4. **The first and second quantities are switched, changing the comparison.** Common flaw: says "it's different" without stating specifically which quantity now comes first. Next move: ask the student to name what each version actually compares.
5. **Total parts 14; one part 3; adults 15 and children 27.** Likely wrong answer: swaps the results, reporting 27 adults and 15 children. Misconception: losing track of which ratio number matches which label. Next move: "does the first number in adults:children describe adults or children?"
6. **Total parts 10; oranges .** Likely wrong answer: (uses the apples part as the denominator instead of the total). Misconception: the recurring reversed-denominator error. Next move: "7 out of how many total parts?"
7. **If 4 parts = 20, one part = 5; other group = 25.** Likely wrong answer: assumes the two groups are equal in size and reports 20 for the other group too. Misconception: not recognizing that a 4:5 ratio means unequal-sized groups. Next move: "does a ratio of 4:5 mean the two groups are the same size?"
8. **Answers vary but must solve from total parts.** Common flaw: writes a real-world scenario but solves it by guessing or estimating instead of using the total-parts method. Next move: check that the total parts, one-part value, and final split are all shown.
9. **Answers vary.** Common flaw: creates a ratio but never states both part-to-whole fractions, or the fractions don't add to 1. Next move: check that the two fractions sum to a whole.
10. **The boxes show equal ratio units and their sum.** Common flaw: says the diagram "helps visualize" without explaining specifically that it verifies the parts add back to the total. Next move: ask the student to point to where the check happens in the diagram.
11. **6 members. The number of meetings per week (2) and the number of weeks (5) were not needed to find the number of members.** Likely wrong answer: works the 2 meetings or 5 weeks into the calculation somehow. Misconception: assuming every number in a word problem must be used. Next move: "what does the question actually ask for? Which numbers answer that?"
12. **Answers vary; must describe a situation where 35 is the total group, and the ratio 2:5 splits it**

into two named parts (10 and 25), with each number's meaning stated. *Common flaw: gives a context but doesn't identify what the 2 and 5 individually represent, or confuses them with the actual quantities (10 and 25). Next move: ask the student to finish the sentences "2 represents ___ parts" and "the actual amount for that part is ___."*

Extend Your Thinking

1. is not red.
2. The ratio stays 2:3 either way — it always has 5 total parts. If the group of 25 doubles to 50, the amount each part represents doubles from 5 to 10, so the actual counts become 20 and 30 instead of 10 and 15.
3. Any ratio with red part-to-whole fraction, such as 2:3 or 4:6.
4. A part-to-whole fraction in a standard positive ratio is at most 1;

a comparison greater than 1 would require a different meaning or reference.

5. 4:6 is 2:3 scaled by 2; 4:5 uses a different relationship.
6. Labels and order determine meaning; the larger number may represent the second quantity, and which quantity is larger depends on the context.

Tripwire and Exit Ticket

Tripwire: total parts 5; boys .

Exit Ticket: red ;

one ratio part 3, so 12 boys and 27 girls; 2:3;

part-to-part requires adding parts before finding a whole fraction;

checks include recombining quantities or matching the tape diagram.

